Unit 9

Differential Equations

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If m and n are the order and degree, respectively of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)^4\right]=0$, then write the value of m + n.

(a) 1 (b) 2 (c) 3 (d) 4

2. The number of solutions of the differential equation $\frac{dy}{dx}=\frac{y+1}{x-1}$, when y(1) = 2 is

(a) one (b) one (c) two (d) infinite

3. The number of arbitrary constants in the particular solution of a differential equation of second order is (are):

(a) 0 (b) 1 (c) 2 (d) 3

4. Kapila is trying to find the general solution of the following differential equations.

(I) $xe^{x/y}dx - ye^{3x/y}dy = 0$

(II) $(2x+1)\frac{dy}{dx} = 3 - 2y\,c$

(III) $\frac{dy}{dx} = \sin x - \cos y$

Which of the above become variable separable by substituting y = b.x, where b is a variable?

(a) only (I) (b) only (I) and (II) (c) all - (I), (II) and (III) (d) None of these

5. The general solution of the differential equation ydx – xdy = 0; (Given x, y > 0), is of the form:

(a) xy = c (b) $x = cy^2$ (c) y = cx (d) $y = cx^2$

(Where 'c' is an arbitrary positive constant of integration)

6. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents:

(a) a circle (b) an ellipse (c) a straight line (d) a parabola

7. The order and the degree of the differential equation $3x^2\left(\frac{d^2y}{dx^2}\right)^3 - 3\left(\frac{dy}{dx}\right)^4 + y = 0$ are:

(a) 2, 1 (b) 2, 3 (c) 2, 4 (d) 3, 1

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8. Integrating factor for the differential equation $(x \log x)\frac{dy}{dx} + y = 2 \log x$ is

(a) log (log x) (b) log x (c) e^x (d) x

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): The solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$ with initial condition x = 1 and y = 1 is x = y.

Reason (R): Separation of variable method can be used to solve the differential equation.

10. Assertion (A): Solution of the differential equation $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$ is

$ye^{\tan^{-1}x} = (\tan^{-1} x - 1)e^{\tan^{-1}x} + C$

Reason (R) : The differential equation of the form $\frac{dy}{dx} + Py = Q$, where P, Q be the functions of x or constant, is a linear type differential equation.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the general solution of the following differential equation: $e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$.

12. Solve : $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$

13. Find the general solution of the differential equation $\frac{dy}{dx} = x - 1 + xy - y$.

14. Solve the following differential equation: $\frac{dy}{dx} = x^3 \cos ecy$, given that y(0) = 0.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. For the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$, find the solution curve passing through the point (1, -1).

16. Find the particular solution of the differential equation: $(1 + e^{2x})dy + (1 + y^2)e^x\,dx = 0$ given that y = 1 when x = 0.

17. Solve the following differential equation: $\frac{dy}{dx} + 2y\tan x = \sin x$, given that y = 0, when $x = \frac{\pi}{3}$.

SECTION – D

Questions 18 carry 5 marks.

18. Solve the following differential equation: $\left(1 + e^{y/x}\right)dy + e^{y/x}\left(1 - \frac{y}{x}\right)dx = 0, (x \neq 0)$

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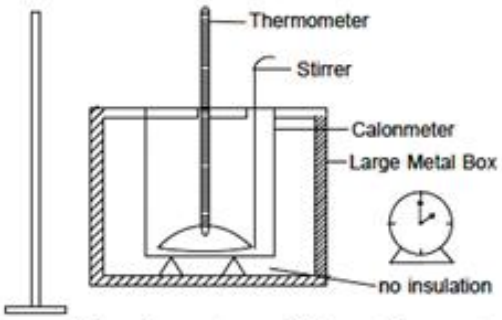
SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.

As per the Newton's law of cooling, the rate at which an object cools is given by the following equation: $\frac{d\theta}{dt} = -k(\theta - \theta_s)$ where $\theta = \theta(t)$ is temperature of cooling object at time, t, θ_s is the temperature of the environment (assumed to be constant) and k is the thermal constant related to the cooling object.



(a) Write the order and degree of the above given differential equation.

(b) Find the solution of the given differential equation: $\frac{d\theta}{dt} = -k(\theta - \theta_s)$

(c) If $\theta = \theta_0$ (initial temperature of cooling object) at time t = 0, then find the particular solution of the above given differential equation.

OR

Find the rate of cooling of the object at 50°C, if the thermal constant is $\frac{3}{25}$ min^{-1} and temperature of surroundings is 25°C.

20. Case-Study 2: Read the following passage and answer the questions given below.

Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50 - y)$ where x denotes the number of weeks and y the number of children who have been given the drops.

(a) Find the solution of the differential equation $\frac{dy}{dx} = k(50 - y)$ (1)

(b) Find the value of c in the particular solution given that y(0) = 0 and k = 0.049 (1)

(c) Find the solution that may be used to find the number of children who have been given the polio drops. (2)